

ADVANCED GCE MATHEMATICS (MEI)

Differential Equations

Candidates answer on the answer booklet.

OCR supplied materials:

- 8 page answer booklet
- (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:

• Scientific or graphical calculator

Wednesday 26 January 2011 Afternoon

Duration: 1 hour 30 minutes

4758/01



INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use g = 9.8.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.
- This document consists of **4** pages. Any blank pages are indicated.

1 (a) The displacement, x m, of a particle at time t seconds is given by the differential equation

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} + 5x = 4\mathrm{e}^t.$$

(i) Find the general solution.

The particle is initially at rest at the origin.

- (ii) Find the particular solution.
- (**b**) The differential equation

$$\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

is to be solved.

(i) Show that 1 is a root of the auxiliary equation and find the other two roots. Hence find the general solution. [5]

When
$$x = 0$$
, $y = 1$ and $\frac{dy}{dx} = -4$. As $x \to \infty$, $y \to 0$.

- (ii) Find the particular solution subject to these conditions. [4]
- (iii) Find the value of x for which y = 0.
- 2 (a) The differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 2xy = \mathrm{e}^{-x^2}\sin x$$

is to be solved subject to the condition x = 0, y = 1.

- (i) Find the particular solution for *y* in terms of *x*.
- (ii) Show that y > 0 for all x and that y has a stationary point when x = 0. State the limiting value of y as |x| → ∞. Hence draw a simple sketch graph of the solution, given that the stationary point at x = 0 is a maximum. [6]
- (b) The differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 2xy = 1$$

is to be solved numerically subject to the condition x = 0, y = 1.

- (i) Use Euler's method with a step length of 0.1 to estimate y when x = 0.2. The algorithm is given by $x_{r+1} = x_r + h$, $y_{r+1} = y_r + hy'_r$. [4]
- (ii) Use the integrating factor method and the approximation $\int_{0}^{0.2} e^{x^2} dx \approx 0.2027$ to estimate y when x = 0.2. [5]

[9]

[4]

[9]

[2]

3 The differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + ky = \cos 3x,$$

where k is a constant, is to be solved.

- (i) Find the complementary function. Hence find the general solution for y in terms of x and k. [8]
- (ii) Find the particular solution subject to the condition that $\frac{dy}{dx} = 1$ when x = 0. [4]

Now consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + ky = 2\mathrm{e}^{-kx}.$$

(iii) Find the general solution.

Now consider the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} = 4\mathrm{e}^{2x}.$$

- (iv) Using your answer to part (iii), or otherwise, solve this differential equation subject to the conditions that y = 0 and $\frac{dy}{dx} = 1$ when x = 0. [6]
- 4 The populations of foxes, x, and rabbits, y, on an island at time t are modelled by the simultaneous differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 0.1x + 0.1y,$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = -0.2x + 0.3y.$$

(i) Show that
$$\frac{d^2x}{dt^2} - 0.4\frac{dx}{dt} + 0.05x = 0.$$
 [5]

- (ii) Find the general solution for *x*. [4]
- (iii) Find the corresponding general solution for y.

Initially there are x_0 foxes and y_0 rabbits.

- (iv) Find the particular solutions.
- (v) In the case $y_0 = 10x_0$, find the time at which the model predicts the rabbits will die out. Determine whether the model predicts the foxes die out before the rabbits. [7]

[6]

[4]

[4]

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.

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Mathematics (MEI)

Advanced GCE

Unit 4758: Differential Equations

Mark Scheme for January 2011

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All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

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1. Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

Μ

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

В

Mark for a correct result or statement independent of Method marks.

Е

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise overspecification.

When a value is given in the paper

Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper

Accept any answer that agrees with the correct value to 2 s.f.

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Mark Scheme

ft should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination.

There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

- i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question		on	Answer	Marks	Guidanc	e
1	(a)	(i)	$\alpha^2 + 2\alpha + 5 = 0$	M1	Auxiliary equation	
			$\alpha = -1 \pm 2j$	A1		
			CF $e^{-t}(A\cos 2t + B\sin 2t)$	M1	CF for complex roots	
				F1	CF for their roots	
			PI $x = ae^t$	B1		
			$\dot{x} = ae^t, \ \ddot{x} = ae^t$	M1	Differentiate twice and substitute	
			$ae^t + 2ae^t + 5ae^t = 4e^t$	M1	Compare coefficients and solve	
			$a = \frac{1}{2}$	A1		
			GS $x = \frac{1}{2}e^{t} + e^{-t}(A\cos 2t + B\sin 2t)$	F1	PI + CF with two arbitrary constants	
			-	[9]		
1	(a)	(ii)	$t = 0, x = 0 \Longrightarrow \frac{1}{2} + A = 0$	M1	Use condition	
			$\dot{x} = \frac{1}{2}\mathbf{e}^t - \mathbf{e}^{-t}(A\cos 2t + B\sin 2t)$	M1	Differentiate (product rule)	
			$+\mathrm{e}^{-t}(-2A\sin 2t+2B\cos 2t)$			
			$t = 0, \ \dot{x} = 0 \Longrightarrow 0 = \frac{1}{2} - A + 2B$	M1	Use condition	
			$x = \frac{1}{2}e^{-t} - \frac{1}{2}e^{-t}(\cos 2t + \sin 2t)$	A1	cao	
				[4]		
1	(b)	(i)	AE $\alpha^3 + 4\alpha^2 + \alpha - 6 = 0$	B1		
			$1^3 + 4 \times 1^2 + 1 - 6 = 0$	E1		
			$(\alpha - 1)(\alpha^2 + 5\alpha + 6) = 0$	M1	Factorise (or solve by other means)	
			$\alpha = 1, -2 \text{ or } -3$	A1		
			$GS y = Ae^x + Be^{-2x} + Ce^{-3x}$	F1	GS = CF with three arbitrary constants	
				[5]		
1	(b)	(ii)	$y \rightarrow 0 \Longrightarrow A = 0$	B1		
			$y(0) = 1 \Longrightarrow B + C = 1$	M1	Use condition	
			$y'(0) = -4 \Longrightarrow -2B - 3C = -4$	M1	Use condition	
			B = -1, C = 2			
			$y = 2e^{-3x} - e^{-2x}$	A1	cao	
				[4]		

Question		n	Answer	Marks	Guidance
1	(b)	(iii)	$y = 0 \Leftrightarrow e^{-3x} (2 - e^x) = 0$		
			$\Leftrightarrow e^x = 2$	M1	
			$\Leftrightarrow x = \ln 2$	A1	
				[2]	
2	(a)	(i)	$I = \exp \int 2x dx$	M1	Attempt IF
			$=e^{x^2}$	A1	Correct IF
			$e^{x^2} \frac{dy}{dx} + 2xe^{x^2} y = \sin x$	M1	Multiply by IF
			$\frac{d}{dx}(ye^{x^2}) = \sin x$	M1	Recognise derivative
			$ye^{x^2} = \int \sin x dx$	M1	Integrate
			$=-\cos x + A$	A1	RHS (including constant)
			$x = 0, y = 0 \Longrightarrow 1 = -1 + A$	M1	Use condition
			$\Rightarrow A = 2$	A1	
			$y = e^{-x^2} (2 - \cos x)$	F1	Divide by their IF, including constant
				[9]	
2	(a)	(ii)	$e^{-x^2} > 0, \cos x \le 1$	M1	
			$\Rightarrow 2 - \cos x > 0 \Rightarrow y > 0$	E1	
			$\frac{dy}{dx} = -2xe^{-x^2}(2 - \cos x) + e^{-x^2}(\sin x)$		Or use DE
			$x = 0 \Rightarrow \frac{dy}{dx} = 0$	E1	
			$ x \to \infty \Longrightarrow y \to 0$	B1	
				B1	Through (0,1)
				B1	Shape consistent with results shown
				[6]	

Question		on	Answer	Marks	Guidance
2	(b)	(i)	$\frac{dy}{dx} = 1 - 2xy$ $x y y'$ $0 1 1$ $0.1 1.1 0.78$ $0.2 1.178$	B1 M1 A1 A1 [4]	First row Use algorithm 1.1 1.178
2	(b)	(ii)	$I = e^{x^{2}}$ $\frac{d}{dx}(ye^{x^{2}}) = e^{x^{2}}$ $\left[ye^{x^{2}}\right]_{x=0}^{x=0.2} = \int_{0}^{0.2} e^{x^{2}} dx$ $y(0.2)e^{0.2^{2}} - 1 = 0.2027$ $y(0.2) = 1.15(55)$	F1 M1 A1 M1 A1 A1 [5]	
3	(i)		$\lambda + k = 0 \Longrightarrow \lambda = -k$ CF Ae^{-kx} PI $y = a\cos 3x + b\sin 3x$ $\frac{dy}{dx} = -3a\sin 3x + 3b\cos 3x$ $-3a\sin 3x + 3b\cos 3x$ $+k(a\cos 3x + b\sin 3x) = \cos 3x$ -3a + kb = 0 3b + ka = 1 $a = \frac{k}{9 + k^2}, b = \frac{3}{9 + k^2}$ $y = Ae^{-kx} + \frac{1}{9 + k^2}(k\cos 3x + 3\sin 3x)$	M1 A1 B1 M1 M1 A1 A1 F1 [8]	Root of auxiliary equation Differentiate Substitute and compare PI + CF with one arbitrary constant

Question		on	Answer	Marks	Guidan	ce
3	(ii)		$x = 0, y' = 1 \Longrightarrow y = 0$ (from DE)	M1A1		
			OR differentiate <i>y</i>			
			$0 = A + \frac{k}{0 + t^2}$	M1	Use condition	
			$9+\kappa$	A1		
			$y = \frac{1}{9+k^2} (k\cos 3x + 3\sin 3x - ke^{-kx})$	7 1 1		
				[4]		
3	(iii)		CF Be^{-kx}	F1		
			PI $y = cxe^{-kx}$	B1		
			$y' = c e^{-kx} - kcx e^{-kx}$	M1	Differentiate	
			$ce^{-kx}(1-kx) + kcxe^{-kx} = 2e^{-kx}$	M1	Substitute and compare	
			<i>c</i> = 2	A1		
			$y = Be^{-kx} + 2xe^{-kx}$	F1	PI + CF with one arbitrary constant	
				[6]		
3	(iv)		$\frac{d}{dx}$ (previous DE) with $k = -2$	M1	Recognise relationship	
			$y = Be^{2x} + 2xe^{2x} + C$	F1		
			$x = 0, y = 0 \Longrightarrow 0 = B + C$	B1	Condition	
			$y' = 2Be^{2x} + 2e^{2x} + 4xe^{2x}$	M1	Differentiate	
			$x = 0, y' = 1 \Longrightarrow 1 = 2B + 2$	M1	Use condition	
			$B = -\frac{1}{2}, C = \frac{1}{2}$		NEED ALTERNATIVE SOLUTION	
			$y = -\frac{1}{2}e^{2x} + 2xe^{2x} + \frac{1}{2}$	A1		
				[6]		
			OR for first 2 marks			
			$m^2 - 2m = 0$; CF $y = Be^{2x} + C$			
			and PI $y = pxe^{2x}$ giving $p = 2$	M1	Complete method	
			$GS y = Be^{2x} + 2xe^{2x} + C$	A1	compress meanod	

Question		on	Answer	Marks	Guidance
4	(i)		$y = 10\dot{x} - x$	M1	
			$\dot{y} = 10\ddot{x} - \dot{x}$	M1	
			$10\ddot{x} - \dot{x} = -0.2x + 3\dot{x} - 0.3x$	M1	Eliminate y
			$\ddot{x} - 0.4\dot{x} + 0.05x = 0$	M1	Eliminate ý
				E1	
	([5]	
4	(ii)		$\lambda^2 - 0.4\lambda + 0.05 = 0$	M1	Auxiliary equation
			$\lambda = 0.2 \pm 0.1 \mathrm{j}$	A1	
			$x = e^{0.2t} (A\cos 0.1t + B\sin 0.1t)$	M1	CF for complex roots
				F1	CF for their roots
1	(:::)			[4]	Differentiate (and dust mla)
4	(III)		$x = 0.2e^{32t} (A\cos 0.1t + B\sin 0.1t)$		Differentiale (product rule)
			$+0.1e^{0.2t}(-A\sin 0.1t + B\cos 0.1t)$		
			$y = 10\dot{x} - x$	M1	Substitute to find <i>y</i>
			$=e^{0.2t}((A+B)\cos 0.1t + (B-A)\sin 0.1t)$	A1	
				[4]	
4	(iv)		$x_0 = A$	B1	
			$y_0 = A + B$	M1	Use condition
			$x = e^{0.2t} (x_0 \cos 0.1t + (y_0 - x_0) \sin 0.1t)$	A1	
			$y = e^{0.2t} (y_0 \cos 0.1t + (y_0 - 2x_0) \sin 0.1t)$	A1	
				[4]	
4	(v)		$y = 0$ when $\tan 0.1t = -y_0 = -1.25$	M1	
			$y = 0$ when $\tan 0.11 = \frac{1}{y_0 - 2x_0} = -1.25$	F1	
			So (for least positive <i>t</i>), $t = 22.5$	A1	Or compare values of tan 0.1 <i>t</i>
			$x = 0$ when $\tan 0.1t = \frac{-x_0}{-x_0} = \frac{1}{-x_0}$	M1	
			$x = 0$ when $\tan 0.11 = \frac{1}{y_0 - x_0} = -\frac{1}{9}$	F1	
			So (for least positive <i>t</i>), $t = 30.3$	A1	Or compare values of tan 0.1 <i>t</i>
			Hence rabbits die out first	A1	Complete argument
				[7]	

4758 Differential Equations (Written Examination)

General Comments

There were many excellent responses to this paper. Candidates seemed familiar with the methods required and were able to employ them appropriately. As is usually the case, most candidates opted to answer questions 1 and 4 together with either question 2 or question 3. The first question was a good starter for almost all candidates and many scored full marks. The other three questions presented some problems, particularly in the later parts. The constant included in the given differential equation in question 3 and the decimals included in the given differential equation 4 were the source of some confusion and numerical errors respectively.

Comments on Individual Questions

- 1) Second order differential equation
- (a)(i) This part of the question was almost always answered correctly.
- (a)(ii) Again, mainly correct answers. Some candidates lost one or two marks through inaccuracy in differentiation or through numerical slips in finding the values of the constant coefficients.
- (b)(i) This part of the question was, again, almost always correctly answered. A small number of candidates factorised the quadratic equation wrongly.
- (b)(ii) The key here was to realise that the coefficient of the exponential term corresponding to the given root of the auxiliary equation had to be zero to satisfy the given boundary conditions. Those who failed to spot this were unable to solve to find the other two coefficients.
- (b)(iii) It was pleasing to see that most candidates were able to solve the exponential equation to obtain $x = \ln 2$.
- 2) First order differential equation
- (a)(i) The vast majority of candidates who attempted this question gained full marks in this part. Occasionally a constant of integration was omitted.
- (a)(ii) This part caused a few problems for many candidates. The simplest way to show that y has a stationary point at x = 0 is to use the given differential equation and substitute the fact that both x and the first derivative of y are zero. The result follows immediately. Unfortunately a significant number of candidates differentiated their expression for y obtained in part (i) and then attempted to solve the resulting equation. This was never a successful approach. The sketch graphs were variable in quality. All that was required was a simple sketch using the information given in this part of the question, namely a curve with a maximum point at (0,1) and that remained positive for all values of x whilst tending to zero for large positive and negative values of x.
- (b)(i) This straightforward example of the use of Euler's method posed few problems for candidates. Occasionally, there was some confusion with the estimation of the value of y for x = 0.3 being offered as the estimate for x = 0.2.

- (b)(ii) Candidates invariably integrated correctly using an integrating factor, but then had difficulty in using the limits of 0 and 0.2 appropriately. Often on the left hand side of their integrated expression, *y* was taken out as a common factor before the limits were applied.
- 3) First order differential equation
- (i) Candidates clearly knew how to attempt this question and almost all found the correct complementary function. The search for the particular integral, however, was not often successful. Candidates became confused between the constant coefficients in their trial particular integral and the constant k in the given differential equation.
- (ii) The method here was applied correctly to what was usually an incorrect general solution from part (i)
- (iii) Many candidates continued with the method they had used in part (i) of this question. Others used the integrating factor method and this latter gave a neat solution.
- (iv) Very few candidates were able to use the approach suggested in the question. They spotted the similarity between the two differential equations, with k = -2, but did not know what to do with this information. Those who opted for the "otherwise" approach of solving the new differential equation from scratch were usually more successful.
- 4) Simultaneous differential equations
- (i) Solutions were almost always fully correct.
- (ii) Again, the vast majority of candidates obtained full marks for a correct general solution for *x*.
- (iii) It was pleasing to see that almost all candidates employed the correct approach here, differentiating their expression for *x* and using it in the first of the given differential equations to find *y*. Unfortunately the decimal quantities involved seemed to lead to more numerical errors than might have been expected. The numbers 0.1, 0.2 and 10 together with some dubious use of brackets led to many incorrect expressions for *y*.
- (iv) By this stage, the number of correct solutions was decreasing fairly rapidly with some extra confusion coming from the manipulation of four constants, and an uncertainty as to which were to appear in the final expressions.
- (v) There were some excellent solutions to this part of the question, from candidates who had negotiated carefully the earlier parts of the question and now knew how to solve the equations involving a sine term and a cosine term. It was unfortunate that a few who had done well thus far used degrees rather than radians. Candidates who had made errors earlier in the question could potentially have gained credit, but the majority failed to recognise a method for extracting *t* from their equations. The easiest approach was to write sin/cos as tan. An alternative was to rewrite their expressions in the form $R\sin(x + \alpha)$.

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